

Wilcoxon Signed Rank Test

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The Wilcoxon Signed Rank Test is a non-parametric statistical test for testing hypothesis on median.

The test has two versions: "single sample" and "paired samples / two samples".



Single Sample

The first version is the analogue of independent one sample t-test ^[1] in the non parametric context. It uses a single sample and is recommended for use whenever we desire to test a hypothesis about population median.

m_0 = the specific value of population median

The null hypothesis ^[2] here is of the form $H_0 : m = m_0$, where m_0 is the specific value of population median that we wish to test against the alternative hypothesis $H_1 : m \neq m_0$.

For example, let us suppose that the manager of a boutique claims that median income his clients is \$24,000/- per annum. To test if this is tenable, the analyst will obtain the yearly income of a sample of his clients and test the null hypothesis $H_0 : m_0 = 24,000$.

Paired Samples

The second version of the test uses paired samples and is the non parametric analogue of dependent t-test for paired samples [3].

This test uses two samples but it is necessary that they should be paired. Paired samples imply that each individual observation of one sample has a unique corresponding member in the other sample.

An Example - Paired Samples

For example, suppose that we have a sample of weights of n obese adults before they are subjected to a change of diet.

After a lapse of six months, we would like to test whether there has been any significant loss in weight as a result of change in diet. One could be tempted to straightaway use the dependent t-test for paired samples [3] here.

However that test has certain assumption notable among them being normality. If this normality assumption is not satisfied, one would have to go for the non parametric Wilcoxon Signed Rank Test [4].

The null hypothesis [2] then would be that there has been no significant reduction in median weight after six months against the alternative that medians before and after significantly differ.

Normality Assumptions is not Required

Most of the standard statistical techniques can be used provided certain standard assumptions such as independence, normality etc. are satisfied.

Often these techniques cannot be used if the normality assumption is not satisfied. Among others, the t-test [5] requires this assumption and it is not advisable to use it if this assumption is violated.

Advantages

The advantage with Wilcoxon Signed Rank Test is that it neither depends on the form of the parent distribution nor on its parameters. It does not require any assumptions about the shape of the distribution.

For this reason, this test is often used as an alternative to t test's whenever the population cannot be assumed to be normally distributed [6]. Even if the normality assumption holds, it has been shown that the efficiency of this test compared to t-test is almost 95%.

Let us illustrate how signed ranks are created in one-sample case by considering the example explained above. Assume that a sample of yearly incomes of 10 customers was collected. The null hypothesis to be tested is $H_0 : m = 24,000$.

We first calculate the deviations of the given observations from 24,000 and then rank them in order of magnitude. This has been done in the following table:

Income	Deviation	Signed Ranks
23,928	-72	-1
24,500	500	5.5
23,880	-120	-2
24,675	675	7
21,965	-2035	-10
22,900	-1100	-9
23,500	-500	-5.5
24,450	450	4
22,998	-1002	-8
23,689	-311	-3

The deviations are ranked in increasing order of absolute magnitude and then the ranks are given the signs of the corresponding deviations.

In the above table the difference 500 occurs twice. In such a case, we assign a common rank which is the arithmetic mean of their respective ranks. Hence 500 was assigned the rank which is the arithmetic mean of 5 and 6.

In a two sample case, the ranks are assigned in a similar way. The only difference is that in a two sample case we first find out the differences between the corresponding observations of the samples and then rank them in increasing order of magnitude.

The ranks are then given the sign of the corresponding differences.

Source URL: <https://explorable.com/wilcoxon-signed-rank-test>

Links

- [1] <https://explorable.com/independent-one-sample-t-test>
- [2] <https://explorable.com/null-hypothesis>
- [3] <https://explorable.com/dependent-t-test-for-paired-samples>
- [4] http://en.wikipedia.org/wiki/Wilcoxon_signed-rank_test
- [5] <https://explorable.com/students-t-test-2>
- [6] <https://explorable.com/normal-probability-distribution>