Normal Probability Distribution

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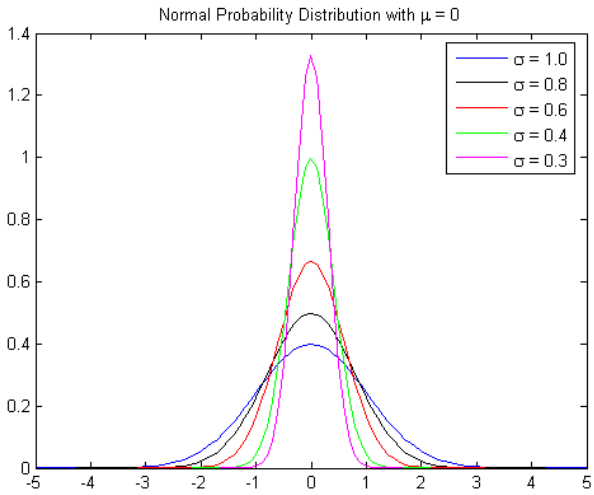
Normal probability distribution, also called Gaussian distribution refers to a family of distributions that are bell shaped.

These are symmetric in nature and peak at the mean, with the probability distribution decreasing away before and after this mean smoothly, as shown in the figure below.

The figure also shows a family of curves with different peaks centered about the same mean, which differ in their spread and height.

? = Mean of the Population
? = Standard Deviation



Normal distribution occurs very frequently in statistics, economics, natural and social sciences and can be used to approximate many distributions occurring in nature and in the manmade world.

For example, the height of all people of a particular race, the length of all dogs of a particular breed, IQ, memory and reading skills of people in a general population and income distribution in an economy all approximately follow the normal probability distribution shaped like a bell curve.

The theory of normal distribution also finds use in advanced sciences like astronomy, photonics and quantum mechanics.

The normal distribution can be characterized by the mean and standard deviation. The mean determines where the peak occurs, which is at 0 in our figure for all the curves. The standard deviation is a measure of the spread of the normal probability distribution, which can be seen as differing widths of the bell curves in our figure.

The Formula

The mean [1] is generally represented by ? and the standard deviation by ?. For a perfect normal distribution, the mean [1], median [2] and mode [3] are all equal. The normal distribution function can be written in terms of the mean and standard deviation as follows:

p(x) = (1/?Root(2?)) x exp (- (x-?)^2/2?^{2})

or

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

(from wikipedia.org)

From the above formula for normal distribution, it can be inferred that about 68% of all values lie within one standard deviation from the mean; 95.4% of all values lie within two standard deviations from the mean and 99.7% of all values lie within three standard deviations from the mean.

From the basic bell curve, there can be many special cases derived that become meaningful under different situations.

For example the left or right or both sides of a normal distribution can be skewed or there could be the presence of long tails.

A basic study of the normal distribution therefore is necessary before a meaningful study can be made into these special cases. This concept can be extended to 3-D normal distributions as well, which are used for more advanced applications.

Links
[1] https://explorable.com/arithmetic-mean
[2] https://explorable.com/calculate-median
[3] https://explorable.com/statistical-mode