Independent Two-Sample T-Test

The independent two-sample t-test is used to test whether population means are significantly different from each other, using the means from randomly drawn samples.

Any statistical test that uses two samples drawn independently of each other and using t-distribution, can be called a 'two-sample t-test'.

Hypothesis Testing

Generally speaking, this test involves testing the null hypothesis [1] H_0: ?(x) = ?(y) against the alternative research hypothesis [2], H_1: ?(x) ? ?(y) where ?(x) and ?(y) are respectively the population mean of the two populations from which the two samples have been drawn.

Hypothesis testing [3] is frequently used for the scientific method.

An Example

Suppose that a school has two buildings - one for girls and the other for boys. Suppose that the principal want to know if the pupils of the two buildings are working equally hard, in the sense that they put in equal number of hours in studies on the average.

Statistically speaking, the principal is interested in testing whether the average number of hours studied by boys is significantly different [4] from the average for girls.

Steps

1. To calculate, we begin by specifying the hypothesis to be tested [3].

   In this case, the null hypothesis [1] would be H_0: ?(boys) = ?(girls), which essentially states that mean study hours for boys and girls are no different.

   The alternative research hypothesis [2] is H_1: ?(boys) ? ?(girls).

2. In the second step, we take a sample of say 10 students from the boy's building and 15 from girl's building and collect data on how long they study daily. These 10 and 15 different study hours are our two samples.

   It is not difficult to see that the two samples have been drawn independent of each other - an essential requirement of the independent two-sample t-test.

   Suppose that the sample mean turns out to be 7.25 hours for boys and 8.5 for girls. We cannot infer anything directly from these sample means - specifically as to whether boys and girls were equally hard working as it could very well have happened by sheer luck (even though the samples were drawn randomly) that boys included in the boy's sample were those who studied fewer hours.

   On the other hand, it could also be the case that girls were indeed working harder than boys.

3. The third step would involve performing the independent two-sample t-test which helps us to either accept or reject the null hypothesis.

   If the null hypothesis is rejected, it means that two buildings were significantly different in terms of number of hours of hard work.

   On the other hand if the null hypothesis is accepted, one can conclude that there is no evidence to suggest that the two buildings differed significantly and that boys and girls can be said to be at par.

Assumptions

Along with the independent single sample t-test [5], this test is one of the most widely tests. However, this test can be used only if the background assumptions are satisfied.

- The populations from which the samples have been drawn should be normal [6] - appropriate statistical methods exist for testing this assumption (For example, the Kolmogorov Smirnov non-parametric test). One needs to note that the normality assumption has to be tested individually and separately for the two samples. It has however been shown that minor departures from normality do not affect this test - this is indeed an advantage.

- The standard deviation [7] of the populations should be equal i.e. ?X2 = ?Y2 = ?2, where ?2 is unknown. This assumption can be tested by the F-test [8].
- Samples have to be randomly drawn independent of each other. There is however no requirement that the two samples should be of equal size - often times they would be unequal though the odd case of equal size cannot be ruled out.

Links
[1] https://explorable.com/null-hypothesis
[2] https://explorable.com/research-hypothesis
[3] https://explorable.com/hypothesis-testing
[4] https://explorable.com/significance-test-2
[5] https://explorable.com/independent-one-sample-t-test
[6] https://explorable.com/normal-probability-distribution
[7] https://explorable.com/calculate-standard-deviation
[8] https://explorable.com/f-test