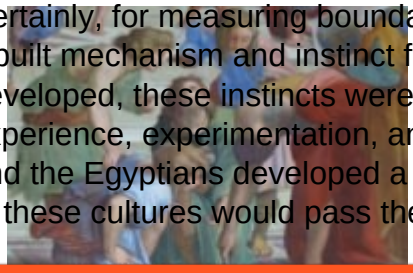
Greek Geometry

Martyn Shuttleworth103.8K reads

Geometry can conceivably lay claim to being the oldest branch of mathematics outside arithmetic, and humanity has probably used geometrical techniques since before the dawn of recorded history. Initially, as with the Egyptians, geometry originated from practical necessity and the need to measure land; the word 'Geometry' means 'Earth Measuring'.



Certainly, for measuring boundaries and for erecting buildings, humans need to have some inbuilt mechanism and instinct for judging distances, angles, and height. As civilizations developed, these instincts were augmented by observations and procedures gained from experience, experimentation, and intuition. The Babylonians were certainly skilled geometers, and the Egyptians developed a rich and complex mathematics based around surveying. Both of these cultures would pass their information on to the Greeks.

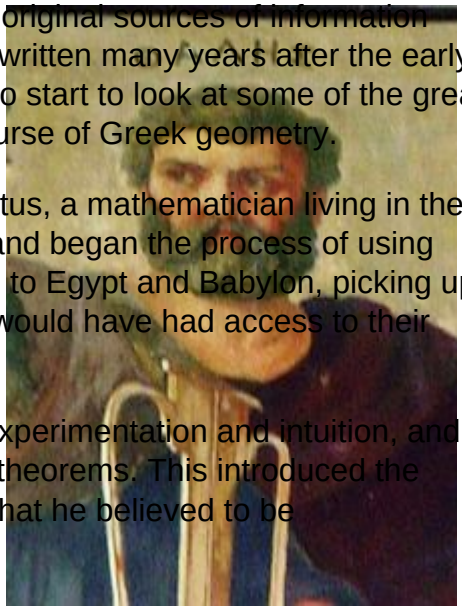
The Start of Greek Geometry

The Egyptians and the Babylonians were not really interested in finding out axioms and underlying principles governing geometry. Their approach was very pragmatic and aimed very much at practical uses. The Babylonians, for example, assumed that Pi was exactly 3, and saw no reason to change this. The Egyptian mathematicians had no structure to their geometry, just a collection of rules and solutions aimed at specific circumstances, such as calculating the volume of a truncated pyramid. They also used trigonometry at that point, in development of a subset of geometry, for surveying and for measuring the dimensions of pyramids.

These cultures did not appear to use deductive reasoning to uncover geometric techniques from first principles. Instead, they used trial and error and, if a solution was not readily available, used trial and error to arrive at an approximation. However, these cultures laid the foundations of Greek geometry and influenced the Greeks, who would bring a deductive methodology to geometry, trying to find elegant rules underpinning the field.

Early Greek Geometry

The early history of Greek geometry is unclear, because no original sources of information remain and all of our knowledge is from secondary sources written many years after the early period. However, we can still see a decent overview and also start to look at some of the great names, the Greek mathematicians who would shape the course of Greek geometry.



Thales of Miletus (Public Domain)

The first, and one of the greatest names, is Thales [1] of Miletus, a mathematician living in the 6th century BCE. He is regarded as the father of geometry and began the process of using deduction from first principles. It is believed that he travelled to Egypt and Babylon, picking up geometric techniques from these cultures, and he certainly would have had access to their work.

Thales strongly believed that reasoning should supersede experimentation and intuition, and began to look for solid principles upon which he could build theorems. This introduced the idea of proof into geometry and he proposed some axioms that he believed to be mathematical truths.

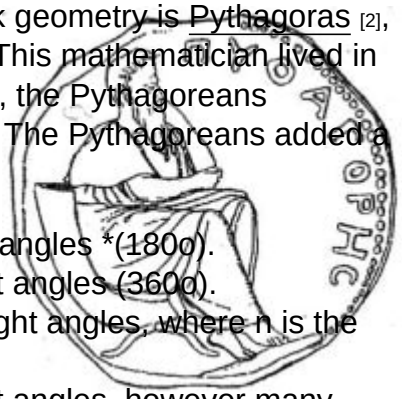
- A circle is bisected by any of its diameters
- The base angles of an isosceles triangle are equal
- When two straight lines cross, the opposing angles are equal
- An angle drawn in a semi-circle is a right angle
- Two triangles with one equal side and two equal angles are congruent

Thales is credited with devising a method for finding the height of a ship at sea, a technique that he used to measure the height of a pyramid, much to the delight of the Egyptians. For this, he had to understand proportion and possibly the rules governing similar triangles, one of the staples of trigonometry and geometry.

It is unclear exactly how Thales decided that the above axioms were irrefutable proofs, but they were incorporated into the body of Greek mathematics and the influence of Thales would influence countless generations of mathematicians.

Pythagoras

Probably the most famous name during the development of Greek geometry is Pythagoras [2], even if only for the famous law concerning right angled triangles. This mathematician lived in a secret society which took on a semi-religious mission. From this, the Pythagoreans developed a number of ideas and began to develop trigonometry. The Pythagoreans added a few new axioms to the store of geometrical knowledge.



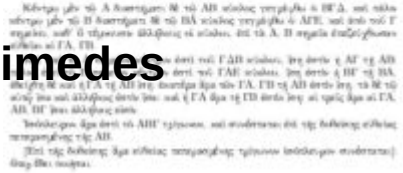
Pythagoras (Public Domain)

- The sum of the internal angles of a triangle equals two right angles *(180o).
- The sum of the external angles of a triangle equals four right angles (360o).
- The sum of the interior angles of any polygon equals 2n-4 right angles, where n is the number of sides.
- The sum of the exterior angles of a polygon equals four right angles, however many sides.
- The three polygons, the triangle, hexagon, and square completely fill the space around a point on a plane - six triangles, four squares and three hexagons. In other words, you can tile an area with these three shapes, without leaving gaps or having overlaps.
- For a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Most of these rules are instantly familiar to most students, as basic principles of geometry and trigonometry. One of his pupils, Hippocrates [3], took the development of geometry further. He was the first to start using geometrical techniques in other areas of maths, such as solving quadratic equations, and he even began to study the process of integration. He studied the problem of Squaring the Circle (which we now know to be impossible, simply because Pi is an irrational number). He solved the problem of Squaring a Lune and showed that the ratio of the areas of two circles equalled the ratio between the squares of the radii of the circles.

Euclid

Alongside Pythagoras, Euclid [5] is a very famous name in the history of Greek geometry. He gathered the work of all of the earlier mathematicians and created his landmark work, 'The Elements,' surely one of the most published books of all time. In this work, Euclid set out the approach for geometry and pure mathematics generally, proposing that all mathematical statements should be proved through reasoning and that no empirical measurements were needed. This idea of proof still dominates pure mathematics in the modern world.



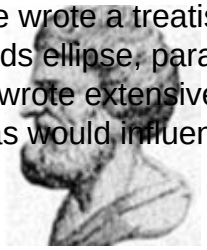
Archimedes

Euclid meant that all mathematical statements should be proved (Euclid's Elements [4])

Archimedes [6] was a great mathematician and was a master at visualising and manipulating space. He perfected the methods of integration and devised formulae to calculate the areas of many shapes and the volumes of many solids. He often used the method of exhaustion to uncover formulae. For example, he found a way to mathematically calculate the area underneath a parabolic curve; calculated a value for Pi more accurately than any previous mathematician; and proved that the area of a circle is equal to Pi multiplied by the square of its radius. He also showed that the volume of a sphere is two thirds the volume of a cylinder with the same height and radius. This last discovery was engraved into his tombstone.

Apollonius of Perga (262 - 190 BCE)

Apollonius was a mathematician and astronomer, and he wrote a treatise called 'Conic Sections.' Apollonius [7] is credited with inventing the words ellipse, parabola, and hyperbola, and is often referred to as the Great Geometer. He also wrote extensively on the ideas of tangents to curves, and his work on conics and parabolas would influence the later Islamic scholars and their work on optics.



Appolonius of Pergia (Public Domain)

Greek Geometry and Its Influence

Greek geometry eventually passed into the hands of the great Islamic scholars, who translated it and added to it. In this study of Greek geometry, there were many more Greek mathematicians and geometers who contributed to the history of geometry, but these names are the true giants, the ones that developed geometry as we know it today.

Links
[1] https://explorable.com/thales
[2] https://explorable.com/pythagoras
[3] http://en.wikipedia.org/wiki/Hippocrates
[4] http://en.wikipedia.org/wiki/Euclid%27s_Elements
[5] https://explorable.com/euclid
[6] https://explorable.com/archimedes
[7] http://en.wikipedia.org/wiki/Apollonius_of_Perga