EXPLORABLE
Think Outside The Box
Published on *Explorable.com* (https://explorable.com)

Geometric Mean

The geometric mean is relevant on certain sets of data, and is different from the arithmetic mean. Mathematically, the geometric mean is the n^{th} root of the product of n numbers.

This can be written as:

Geometric Mean = (a1 × a2 . . . an)^1/n

Where:

N = Number of datapoints

a = score of a datapoint

or more complicated - the exact formula (from wikipedia [1])

$$\left(\prod_{i=1}^{n} a_i\right)^{1/n} = \sqrt[n]{a_1 a_2 \cdots a_n}$$

The geometric mean is relevant on those sets of data [2] that are products or exponential in nature. This includes a variety of branches of natural sciences and social sciences.

Examples

For example, if a strain of bacteria increases its population by 20% in the first hour, 30% in the next hour and 50% in the next hour, we can find out an estimate of the mean percentage growth in population.

In this case, it is the geometric mean, and not the arithmetic mean that is relevant. To see this, start off with 100 bacteria.

- After the first hour, they grow to 120 bacteria, which is a grow rate of 1,2 (100*1.2);
- After the second hour, they grow to 156 bacteria, which is a grow rate of 1,3 (120*1.3)
- After the third hour, they grow to 234 bacteria, which is a grow rate of 1,5 (156*1.5).

Now, we would like to find the mean growth rate

> Geometric Mean = (a1 × a2 . . . an)^1/n
>
> Geometric Mean = (1.2 × 1.3 x 1.5)^1/3
>
> Geometric Mean = (1.2 × 1.3 x 1.5)^1/3
>
> Geometric Mean = (2,34)^1/3
>
> Geometric Mean ? 1.3276

If we find the geometric mean of 1.2, 1.3 and 1.5, we get 1.3276. This should be interpreted as the mean rate of growth of the bacteria over the period of 3 hours, which means if the strain of bacteria grew by 32.76% uniformly over the 3 hour period, then starting with 100 bacteria, it would reach 234 bacteria in 3 hours.

Therefore whenever we have percentage growth over a period of time, it is the geometric mean and not the arithmetic mean [3] that makes sense.

Usages

In social sciences, we frequently encounter this in a number of ways. For example, the human population growth is expressed as a percentage, and thus when population growth needs to be averaged, it is the geometric mean that is most relevant.

In surveys [4] and studies too, the geometric mean becomes relevant. For example, if a survey found that over the years, the economic status of a poor neighborhood is getting better, they need to quote the geometric mean of the development, averaged over the years in which the survey was conducted. The arithmetic mean will not make sense in this case either.

In economics, we see the percentage growth in interest accumulation. Thus if you are starting out with a sum of money that is compounded for interest, then the mean that you should look for is the geometric mean. Many such financial instruments like bonds yield a fixed percentage return, and while quoting their “average” return, it is the geometric mean that should be quoted.

Source URL: https://explorable.com/geometric-mean

Links:
[1] http://en.wikipedia.org/wiki/Geometric_mean, [2] https://explorable.com/statistical-data-sets, [3] https://explorable.com/arithmetic-mean, [4] https://explorable.com/survey-research-design, [5] https://explorable.com/users/siddharth, [6] https://explorable.com/geometric-mean