Factorial Anova

Experiments where the effects of more than one factor are considered together are called 'factorial experiments' and may sometimes be analyzed with the use of factorial ANOVA.

For instance, the academic achievement of a student depends on study habits of the student as well as home environment. We may have two simple experiments, one to study the effect of study habits and another for home environment.

Independence of Factors

But these experiments [1] will not give us any information about the dependence or independence of the two factors, namely study habit and home environment.

In such cases, we resort to Factorial ANOVA which not only helps us to study the effect of two or more factors but also gives information about their dependence or independence in the same experiment. There are many types of factorial designs like 22, 23, 32 etc. The simplest of them all is the 22 or 2 x 2 experiment.

An Example

In these experiments, the factors are applied at different levels. In a 2 x 2 factorial design,

there are 2 factors each being applied in two levels.

Let us illustrate this with the help of an example. Suppose that a new drug has been developed to control hypertension.

We want to test the effect of quantity of the drug taken and the effect of gender. Here, the quantity of the drug is the first factor and gender is the second factor (or vice versa).

Suppose that we consider two quantities, say 100 mg and 250 mg of the drug (1 / 2). These two quantities are the two levels of the first factor.

Similarly, the two levels of the second factor are male and female (A / B).

Thus we have two factors each being applied at two levels. In other words, we have a 2 x 2 factorial design.

Here we have 4 different treatment groups, one for each combination of levels of factors - by convention, the groups are denoted by A1, A2, B1, B2. These groups mean the following.

- A1 : 100mg of the drug applied on male patients
- A2 : 250mg of the drug applied on male patients
- B1 : 100mg of the drug applied on female patients
- B2 : 250mg of the drug applied on female patients.

Here, the quantity of the drug and gender are the independent variables [2] whereas reduction of hypertension after one month is the dependent variable [3].

Main Effects and Interaction

A main effect is an outcome that can show consistent difference between levels of a factor.

In our example, there are two main effects - quantity and gender.

Factorial ANOVA also enables us to examine the interaction effect between the factors. An interaction effect is said to exist when differences on one factor depend on the level of other factor.

However, it is important to remember that interaction is between factors and not levels. We know that there is no interaction between the factors when we can talk about the effect of one factor without mentioning the other factor.

Hypothesis Testing

In the above example, there are three hypotheses to be tested [4]. These are:

H01: Main effect 'quantity' is not significant

H02: Main effect 'gender' is not significant

H03: Interaction effect is not present.

For main effect gender, the null hypothesis means that there is no significant difference in reduction of hypertension in males and females.

The null hypothesis [5] for the main effect quantity means that there is no significant difference [6] in reduction of hypertension whether the patients are given 100 mg or 250 mg of the drug.

For the interaction effect, the null hypothesis means that the two main effects gender and quantity are independent. The computational aspect involves computing F-statistic for each hypothesis.

Advantages

Factorial design has several important features.

- Factorial designs are the ultimate designs of choice whenever we are interested in examining treatment variations.
- Factorial designs are efficient. Instead of conducting a series of independent studies, we are effectively able to combine these studies into one.
- Factorial designs are the only effective way to examine interaction effects.

The assumptions remain the same as with other designs - normality, independence and equality of variance [7].

Links
[1] https://explorable.com/experimental-research
[2] https://explorable.com/independent-variable
[3] https://explorable.com/dependent-variable
[4] https://explorable.com/hypothesis-testing
[5] https://explorable.com/null-hypothesis
[6] https://explorable.com/significance-test-2
[7] https://explorable.com/statistical-variance