Dependent T-Test for Paired Samples

The dependent t-test for paired samples is used when the samples are paired. This implies that each individual observation of one sample has a unique corresponding member in the other sample.

- one sample has been tested twice (repeated measures)

or,

- two samples have been "matched" or "paired", in some way. (matched subjects design)

The emphasis being on pairing of observations, it is obvious that the samples are dependent - hence the name.

Any statistical test involving paired samples and using t-distribution can be called 't-test for paired samples'.

An Example

Let us illustrate the meaning of a paired sample. Suppose that we are required to examine if a newly developed intervention program for disadvantaged students has an impact. For this purpose, we need to obtain scores from a sample of n such students in a standardized test before administering the program.

After the program is over, the same test needs to be administered to the same group of students and scores obtained again.

There are two samples: 1) the sample of prior intervention scores (pretest) and, 2) the post intervention scores (posttest). The samples are related in the sense that each pretest has a corresponding posttest as both were obtained from the same student.

If the score of each student (ith) before and after the program is xi and yi respectively, then the pair (xi, yi) corresponds to the same subject (student in this case).

This is what is meant by paired sample. It is very important that two scores for each individual student be correctly identified and labeled as the differences di =? xi - yi ?are used to determine the test statistic and consequently the p-value [1].

Steps

1. With the above framework, the null hypothesis [2] would be H0: there is no significant [3] difference between pre and post intervention scores, which essentially states that the intervention program was not effective. The alternative hypothesis [4] is H1: there is significant difference between pre and post intervention scores.
2. Once the hypotheses have been framed, the second step involves taking the sample of pre and post intervention scores and determining the sum, ?? xi - yi ?. Logically speaking, a small sum could indicate truth of the null hypothesis.

   However nothing concrete can be interpreted from it - specifically as to whether intervention program did have an impact as it could very well have happened by sheer luck (even though the students were drawn randomly) that for this sample of students, the scores did not change much.

   On the other hand, it could also be the case that the program was indeed useful.

3. The third step involves performing the dependent t-test for paired samples which helps us to either accept or reject the null hypothesis. If the null hypothesis is rejected, one can infer that the program was useful.

   On the other hand if the null hypothesis is accepted, one can conclude that there is no evidence to suggest the program did have an impact.

Assumptions

This test has a few background assumptions which need to be satisfied.

1. The sample of differences (di's) should be normal - an assumption that can be tested - for instance by the Kolmogorov Smirnov non-parametric test.

   It has however been shown that minor departures from normality do not affect this test - this is indeed an advantage.

2. The samples should be dependent and it should be possible to identify specific pairs.
3. An obvious requirement is that the two samples should be of equal size.

For Small Samples

This test is a small sample test. It is difficult to draw the clearest line of demarcation between large and small sample [5].

Statisticians have generally agreed that a sample may be considered small if its size is < 30 (below 30).

Links
[1] https://explorable.com/p-value
[2] https://explorable.com/null-hypothesis
[3] https://explorable.com/significance-test-2
[4] https://explorable.com/research-hypothesis
[5] https://explorable.com/what-is-sampling