Arithmetic Mean

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The arithmetic mean is perhaps the most commonly used statistical mean to measure the central tendency of data.

The arithmetic mean is also called the "average." It is used in most scientific experiments.

Mathematically, the arithmetic mean is given simply by:

$$\text{arithmetic mean} = \frac{\text{Sum of all datapoints}}{\text{Total number of data-points (n)}}$$

or in a more complicated form (wikipedia):

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n}(x_1 + \cdots + x_n)$$

Examples

If there are three numbers in a data-set, add them and divide by three:

$$\text{arithmetic mean} = \frac{x_1 + x_2 + x_3}{3}$$

Or if there are four numbers, add them and divide by 4:

$$\text{arithmetic mean} = \frac{x_1 + x_2 + x_3 + x_4}{4}$$

For example, the time in seconds taken for a particular chemical reaction under the same laboratory conditions might give values of 11.6, 12.1, 11.8, 11.5 and 12.0.

$$\text{arithmetic mean} = \frac{\text{Sum of datapoints}}{\text{Total number of data-points}}$$

$$\text{arithmetic mean} = \frac{11.6 + 12.1 + 11.8 + 11.5 + 12.0 \text{ s}}{5}$$

$$\text{arithmetic mean} = 11.8 \text{ s}$$

The arithmetic mean of these numbers is 11.8 s.

This would represent the average time for the chemical reaction. If the laboratory conditions were the same, however, why were there differences in reactions times? These can likely be attributed to random errors [1] like random fluctuations in temperature and humidity in the laboratory.

When the word "mean" is used, it generally refers to the arithmetic mean.The mean gives very useful information in cases where the data [2] is relatively symmetric. For example, if the data is nearly normally distributed [3], then the mean is the best measure of central tendency [4]. However, if the data is very skewed, then the arithmetic mean might become misleading.

For example, business schools will often boast about the average placement salaries of their latest batch of students, or newspapers will report on the "average cost of a wedding." While data skewed by a few very high salaries or very expensive weddings will give a true arithmetic average, it's an average which tells us less than we'd like about the general tendencies of the group.

Is the Arithmetic Mean Misleading?

When the arithmetic mean is used to calculate this, it can be misleading because the salaries can be widely spread. Thus it may happen that 10% of the class have gotten excellent job offers while half the class is without jobs. If you were a student trying to understand what you

could expect for your own placement salary, the median [5] would be a much better measure to look at.

However, in some cases, even when the data is skewed, the arithmetic mean does give some valuable information about the data. But it needs to be interpreted in the right manner.

The Gross Domestic Product or GDP used in economics to determine the financial well being of a country is an arithmetic mean. The total value of goods and services produced in the country when averaged out over the total population [6] gives a measure of GDP. The GDP tells us nothing about the *distribution* of wealth inside the country, but can be a good parameter for the country as a whole to work with in improving the economic condition of its citizens.

Similary, the arithmetic mean of wedding cost above may not be useful for a single couple, but might be for a historian who wants to track the changes in this figure over time - or indeed wedding retailers who want to give a false impression of how much a wedding should cost!

Links
[1] https://explorable.com/random-error
[2] https://explorable.com/statistical-data-sets
[3] https://explorable.com/normal-probability-distribution
[4] https://explorable.com/measures-of-central-tendency
[5] https://explorable.com/calculate-median
[6] https://explorable.com/research-population